

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Mathematics-I

Subject Code : 4SC01MTC1

Branch: B. Sc.(All)

Semester : 1

Date :25/04/2016

Time : 10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1 Attempt the following questions: (14)**
- a) Find the eigen values of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$. (02)
- b) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$. (02)
- c) Find the center and radius of the sphere $x^2 + y^2 + z^2 - 4x - 2y - 6z - 11 = 0$. (02)
- d) Check whether the following differential equation is exact or not: (02)
 $(1 + 2xy \cos x^2 - 2xy)dx + (\sin x^2 - x^2)dy = 0$.
- e) Express the equation $x^2 + y^2 = 1$ into the polar form. (02)
- f) Write down the series expansion of e^x in powers of x . (02)
- g) State Roll's mean value theorem. (02)

Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions (14)**
- a) State Cayley-Hamilton theorem and verify it for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Also find A^{-1} . (07)
- b) State and prove Leibnitz's theorem. (07)
- Q-3 Attempt all questions (14)**
- a) Test for consistency and solve (07)
 $5x + 3y + 7z = 4$; $3x + 26y + 2z = 9$; $7x + 2y + 10z = 5$.
- b) State and prove Cauchy mean value theorem. (07)
- Q-4 Attempt all questions (14)**
- a) State standard form of a linear equation of the first order and solve (07)
 $\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$.
- b) Apply Gauss elimination method to solve the equations (07)



$$x + 4y - z = -5; \quad x + y - 6z = -12; \quad 3x - y - z = 4.$$

Q-5 **Attempt all questions** (14)

a) Expand $\log x$ in powers of $(x - 1)$ and hence evaluate $\log 1.1$ correct to four decimal places. (07)

b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (07)

Q-6 **Attempt all questions** (14)

a) If $y = e^{a \sin^{-1} x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. Hence find the value of y_n when $x = 0$. (07)

b) Solve the following system of equations using Cramer's rule:
 $3x + y + 2z = 3; \quad 2x - 3y - z = -3; \quad x + 2y + z = 4$. (07)

Q-7 **Attempt all questions** (14)

a) Define: rank of the matrix and find the rank of matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$. (07)

b) Define: Hermitian matrix. If $A = \begin{bmatrix} 2 + i & 3 & -1 + 3i \\ -5 & i & 4 - 2i \end{bmatrix}$, show that AA^* is a Hermitian matrix, where A^* is conjugate transpose of A . (07)

Q-8 **Attempt all questions** (14)

a) Solve: $(x^2 - y^2)dx - xy dy = 0$. (07)

b) Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form. (07)

